Examples from Multilevel Software Comparative Reviews

Douglas Bates
R Development Core Team
Douglas.Bates@R-project.org

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Abstract

The Center for Multilevel Modelling at the Institute of Education, London maintains a web site of "Software reviews of multilevel modeling packages". The data sets discussed in the reviews are available at this web site. We have incorporated these data sets in the `lme4` package for R and, in this vignette, provide the results of fitting several models to these data sets.

1 Introduction

2 Two-level normal models

The `Exam` data set is used in fitting examples of two-level normal multilevel models.

```
> str(Exam)
`data.frame':        4059 obs. of  10 variables:
 $ school  : Factor w/ 65 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ normexam: num   0.261  0.134 -1.724  0.968  0.544 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ schavg  : num  0.166 0.166 0.166 0.166 0.166 ...
 $ vr      : Factor w/ 3 levels "bottom 25%","mid 50%",..: 2 2 2 2 2 2 2 2 2 2 ...
 $ intake  : Factor w/ 3 levels "bottom 25%","mid 50%",..: 1 2 3 2 2 1 3 2 2 3 ...
 $ standLRT: num   0.619  0.206 -1.365  0.206  0.371 ...
 $ sex     : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
 $ type    : Factor w/ 2 levels "Mxd","Sngl": 1 1 1 1 1 1 1 1 1 1 ...
 $ student : Factor w/ 650 levels "1","2","3","4",..: 143 145 142 141 138 155 158 115 117 113 ...
```

```
> system.time(Em1 <- lmer(normexam ~ standLRT + sex + schgend +
+     (1 | school), Exam), gc = TRUE)
[1] 0.16 0.00 0.16 0.00 0.00
> summary(Em1)
Linear mixed-effects model fit by REML
Formula: normexam ~ standLRT + sex + schgend + (1 | school)
   Data: Exam
      AIC      BIC    logLik MLdeviance REMLdeviance
 9361.673 9405.834 -4673.837   9325.501     9347.673
Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 0.085829 0.29297
 Residual             0.562534 0.75002
# of obs: 4059, groups: school, 65

Fixed effects:
               Estimate  Std. Error   DF t value  Pr(>|t|)
(Intercept)  -1.0493e-03  5.5569e-02 4054 -0.0189   0.98494
standLRT      5.5975e-01  1.2450e-02 4054 44.9601 < 2.2e-16
sexM         -1.6739e-01  3.4100e-02 4054 -4.9089 9.519e-07
schgendboys   1.7769e-01  1.1347e-01 4054  1.5659   0.11745
schgendgirls  1.5900e-01  8.9403e-02 4054  1.7784   0.07541

Correlation of Fixed Effects:
            (Intr) stnLRT sexM   schgndb
standLRT    -0.014
sexM        -0.316  0.061
schgendboys -0.395 -0.003 -0.145
schgendgrls -0.622  0.009  0.197  0.245
```

There are some interesting aspects of data management that show up in the analysis of these data. The `student` variable is an identifier of the student within the `school`. It would be best to combine the indicators of school and student to get a unique identifier of the student.

```
> Exam$ids <- with(Exam, school:student)[, drop = TRUE]
> str(Exam)
`data.frame':        4059 obs. of  11 variables:
 $ school  : Factor w/ 65 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ normexam: num   0.261  0.134 -1.724  0.968  0.544 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ schavg  : num  0.166 0.166 0.166 0.166 0.166 ...
 $ vr      : Factor w/ 3 levels "bottom 25%","mid 50%",..: 2 2 2 2 2 2 2 2 2 2 ...
 $ intake  : Factor w/ 3 levels "bottom 25%","mid 50%",..: 1 2 3 2 2 1 3 2 2 3 ...
 $ standLRT: num   0.619  0.206 -1.365  0.206  0.371 ...
 $ sex     : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
 $ type    : Factor w/ 2 levels "Mxd","Sngl": 1 1 1 1 1 1 1 1 1 1 ...
 $ student : Factor w/ 650 levels "1","2","3","4",..: 143 145 142 141 138 155 158 115 117 113 ...
 $ ids     : Factor w/ 4055 levels "1:1","1:4","1:6",..: 48 49 47 46 45 50 51 39 40 38 ...
```

Notice that there are 4059 observations but only 4055 unique levels of student within school. We can check the ones that are duplicated

```
> Exam$ids[which(duplicated(Exam$ids))]
```

```
[1] 43:86 50:39 52:2  52:21
4055 Levels: 1:1 1:4 1:6 1:7 1:13 1:14 1:16 1:17 1:19 1:22 1:27 ... 65:155
```

One of these duplicated cases is particularly interesting. One of the students with the duplicated student id 86 in school 43 is the only male student in this mixed school. This is probably a case of a misrecorded school.

3 Three-level Normal Models

These results are from the 1997 A-level Chemistry exam. The `school` is nested in `lea` (local education authority) and has unique levels for each of the 2410 schools. It is a good practice to make the nesting explicit by specifying the grouping factors as the 'outer' factor, `lea` in this case, and the interaction of the outer and inner factors, `lea:school` or `school:lea` in this case. This will ensure unique levels for each `school` within `lea` combination.

To fit the model `mC2` we increase the number of EM iterations from its default of 20 to 40. Without this change the current version of the `optim` function in R will declare convergence to an incorrect optimum. By increasing the number of EM iterations we are able to get closer to the optimum before calling `optim` and converge to the correct value. The optim function will be patched so this change will not be needed in future versions of R.

Data from the 1997 A-level Chemistry exam are available as `Chem97`.

```
> str(Chem97)
```

```
`data.frame':        31022 obs. of  8 variables:
 $ lea      : Factor w/ 131 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ school   : Factor w/ 2410 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ student  : Factor w/ 31022 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 10 ...
 $ score    : num  4 10 10 10 8 10 6 8 4 10 ...
 $ gender   : Factor w/ 2 levels "M","F": 2 2 2 2 2 2 2 2 2 2 ...
 $ age      : num  3 -3 -4 -2 -1 4 1 4 3 0 ...
 $ gcsescore: num  6.62 7.62 7.25 7.50 6.44 ...
 $ gcsecnt  : num  0.339 1.339 0.964 1.214 0.158 ...
```

```
> system.time(mC1 <- lmer(score ~ 1 + (1 | lea:school) + (1 |
+     lea), Chem97), gc = TRUE)
```

```
[1] 4.58 0.12 4.70 0.00 0.00
```

```
> summary(mC1)
```

```
Linear mixed-effects model fit by REML
Formula: score ~ 1 + (1 | lea:school) + (1 | lea)
   Data: Chem97
      AIC      BIC   logLik MLdeviance REMLdeviance
 157881.8 157915.2 -78936.9   157869.9     157873.8
Random effects:
 Groups     Name        Variance Std.Dev.
 lea:school (Intercept) 2.74878  1.6579
```

```
 lea        (Intercept) 0.15351  0.3918
 Residual               8.51607  2.9182
# of obs: 31022, groups: lea:school, 2410; lea, 131

Fixed effects:
             Estimate Std. Error    DF t value  Pr(>|t|)
(Intercept) 5.3190e+00 5.8110e-02 31021  91.533 < 2.2e-16
> system.time(mC2 <- lmer(score ~ gcsecnt + (1 | school) +
+     (1 | lea), Chem97, control = list(niterEM = 40)), gc = TRUE)
[1] 2.26 0.03 2.30 0.00 0.00
> summary(mC2)
Linear mixed-effects model fit by REML
Formula: score ~ gcsecnt + (1 | school) + (1 | lea)
   Data: Chem97
    AIC      BIC   logLik MLdeviance REMLdeviance
 141707 141748.7 -70848.5   141685.6       141697
Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 1.166190 1.07990
 lea      (Intercept) 0.014754 0.12147
 Residual             5.154206 2.27029
# of obs: 31022, groups: school, 2410; lea, 131

Fixed effects:
             Estimate Std. Error    DF t value  Pr(>|t|)
(Intercept) 5.6355e+00 3.1232e-02 31020  180.44 < 2.2e-16
gcsecnt     2.4726e+00 1.6904e-02 31020  146.27 < 2.2e-16

Correlation of Fixed Effects:
        (Intr)
gcsecnt 0.058
```

4 Two-level models for binary data

The data frame `Contraception` provides data from the Bangladesh fertility survey.

```
> str(Contraception)
`data.frame':        1934 obs. of  6 variables:
 $ woman   : Factor w/ 1934 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 10 ...
 $ district: Factor w/ 60 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ use     : Factor w/ 2 levels "N","Y": 1 1 1 1 1 1 1 1 1 1 ...
 $ livch   : Factor w/ 4 levels "0","1","2","3+": 4 1 3 4 1 1 4 4 2 4 ...
 $ age     : num   18.44  -5.56   1.44   8.44 -13.56 ...
 $ urban   : Factor w/ 2 levels "N","Y": 2 2 2 2 2 2 2 2 2 2 ...
> summary(Contraception[, -1])
    district    use      livch         age              urban
 14     : 118   N:1175   0 :530   Min.   :-13.560000   N:1372
 1      : 117   Y: 759   1 :356   1st Qu.: -7.559900   Y: 562
 46     :  86            2 :305   Median : -1.559900
 25     :  67            3+:743   Mean   :  0.002198
 6      :  65                     3rd Qu.:  6.440000
 30     :  61                     Max.   : 19.440000
 (Other):1420
```

```
> system.time(mB1 <- lmer(use ~ urban + age + livch + (1 |
+     district), Contraception, family = binomial))
stack imbalance in .Call, 141 then 142
stack imbalance in <-, 135 then 136
stack imbalance in {, 133 then 134
stack imbalance in standardGeneric, 125 then 126
stack imbalance in <-, 119 then 120
stack imbalance in .Internal, 114 then 115
stack imbalance in {, 108 then 109
stack imbalance in .Internal, 99 then 100
stack imbalance in {, 96 then 97
stack imbalance in if, 94 then 95
stack imbalance in {, 92 then 93
stack imbalance in <-, 86 then 87
[1] 0.28 0.01 0.28 0.00 0.00
stack imbalance in {, 84 then 85
stack imbalance in if, 82 then 83
> summary(mB1)
Generalized linear mixed model fit using PQL
Formula: use ~ urban + age + livch + (1 | district)
   Data: Contraception
 Family: binomial(logit link)
      AIC      BIC    logLik deviance
 2401.664 2368.260 -1206.832 2413.664
Random effects:
     Groups        Name    Variance    Std.Dev.
   district (Intercept)     0.21518     0.46387
# of obs: 1934, groups: district, 60

Estimated scale (compare to 1)  0.9844111

Fixed effects:
             Estimate Std. Error  z value  Pr(>|z|)
(Intercept) -1.6606460  0.1452147 -11.4358 < 2.2e-16
urbanY       0.7193097  0.1183317   6.0788 1.211e-09
age         -0.0261558  0.0078152  -3.3468 0.0008176
livch1       1.0921026  0.1565011   6.9782 2.989e-12
livch2       1.3545533  0.1729641   7.8314 4.824e-15
livch3+      1.3241531  0.1773558   7.4661 8.262e-14

Correlation of Fixed Effects:
        (Intr) urbanY age    livch1 livch2
urbanY  -0.300
age      0.446 -0.046
livch1  -0.589  0.059 -0.211
livch2  -0.631  0.094 -0.378  0.488
livch3+ -0.748  0.098 -0.674  0.539  0.619
> system.time(mB2 <- lmer(use ~ urban + age + livch + (1 |
+     district), Contraception, family = binomial, method = "Laplace"))
stack imbalance in .Call, 141 then 142
stack imbalance in <-, 135 then 136
stack imbalance in .Call, 159 then 160
stack imbalance in .Call, 160 then 161
stack imbalance in .Call, 161 then 162
stack imbalance in .Call, 162 then 163
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stack imbalance in .Call, 562 then 563
stack imbalance in .Call, 563 then 564
stack imbalance in .Call, 564 then 565
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stack imbalance in .Call, 574 then 575
stack imbalance in .Call, 575 then 576
stack imbalance in .Call, 576 then 577
stack imbalance in .Call, 577 then 578
stack imbalance in .Call, 578 then 579
stack imbalance in .Call, 579 then 580
stack imbalance in .Call, 580 then 581
stack imbalance in .Call, 152 then 574
stack imbalance in {, 150 then 572
stack imbalance in <-, 144 then 566
stack imbalance in {, 142 then 564
stack imbalance in if, 140 then 562
stack imbalance in {, 138 then 560
stack imbalance in if, 136 then 558
stack imbalance in {, 133 then 556
stack imbalance in standardGeneric, 125 then 548
stack imbalance in <-, 119 then 542
stack imbalance in .Internal, 114 then 537
stack imbalance in {, 108 then 531
stack imbalance in .Internal, 99 then 522
stack imbalance in {, 96 then 519
stack imbalance in if, 94 then 517
stack imbalance in {, 92 then 515
stack imbalance in <-, 86 then 509
[1] 8.38 0.01 8.40 0.00 0.00
stack imbalance in {, 84 then 507
stack imbalance in if, 82 then 505
> summary(mB2)
Generalized linear mixed model fit using Laplace
Formula: use ~ urban + age + livch + (1 | district)
   Data: Contraception
 Family: binomial(logit link)
      AIC      BIC    logLik deviance
 2401.662 2368.258 -1206.831 2413.662
Random effects:
     Groups        Name    Variance    Std.Dev.
   district (Intercept)     0.21518     0.46387
# of obs: 1934, groups: district, 60

Estimated scale (compare to 1)  0.9844111

Fixed effects:
             Estimate Std. Error  z value  Pr(>|z|)
(Intercept) -1.6599219  0.1452147 -11.4308 < 2.2e-16
urbanY       0.7196889  0.1183317   6.0820 1.187e-09
```

```
age         -0.0261433  0.0078152  -3.3452 0.0008223
livch1       1.0917943  0.1565011   6.9763 3.031e-12
livch2       1.3541641  0.1729641   7.8292 4.911e-15
livch3+      1.3235276  0.1773558   7.4626 8.486e-14

Correlation of Fixed Effects:
        (Intr) urbanY age    livch1 livch2
urbanY  -0.300
age      0.446 -0.046
livch1  -0.589  0.059 -0.211
livch2  -0.631  0.094 -0.378  0.488
livch3+ -0.748  0.098 -0.674  0.539  0.619
> system.time(mB3 <- lmer(use ~ urban + age + livch + (urban |
+     district), Contraception, family = binomial))
stack imbalance in .Call, 141 then 142
stack imbalance in <-, 135 then 136
stack imbalance in {, 133 then 134
stack imbalance in standardGeneric, 125 then 126
stack imbalance in <-, 119 then 120
stack imbalance in .Internal, 114 then 115
stack imbalance in {, 108 then 109
stack imbalance in .Internal, 99 then 100
stack imbalance in {, 96 then 97
stack imbalance in if, 94 then 95
stack imbalance in {, 92 then 93
stack imbalance in <-, 86 then 87
[1] 0.53 0.01 0.53 0.00 0.00
stack imbalance in {, 84 then 85
stack imbalance in if, 82 then 83
> summary(mB3)
Generalized linear mixed model fit using PQL
Formula: use ~ urban + age + livch + (urban | district)
   Data: Contraception
 Family: binomial(logit link)
      AIC      BIC    logLik deviance
 2391.121 2368.852 -1199.561 2399.121
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 district (Intercept) 0.38774  0.62269
          urbanY      0.66745  0.81698  -0.793
# of obs: 1934, groups: district, 60

Estimated scale (compare to 1)  0.9759564

Fixed effects:
              Estimate Std. Error  z value  Pr(>|z|)
(Intercept) -1.6665200  0.1572532 -10.5977 < 2.2e-16
urbanY       0.7914232  0.1681257   4.7073 2.510e-06
age         -0.0258502  0.0079082  -3.2688   0.00108
livch1       1.0987723  0.1580051   6.9540 3.550e-12
livch2       1.3342511  0.1745854   7.6424 2.132e-14
livch3+      1.3227367  0.1795440   7.3672 1.743e-13

Correlation of Fixed Effects:
        (Intr) urbanY age    livch1 livch2
urbanY  -0.481
```

```
age      0.416 -0.036
livch1  -0.548  0.038 -0.211
livch2  -0.586  0.068 -0.378  0.487
livch3+ -0.695  0.062 -0.674  0.537  0.616
> system.time(mB4 <- lmer(use ~ urban + age + livch + (urban |
+     district), Contraception, family = binomial, method = "Laplace"))
stack imbalance in .Call, 141 then 142
stack imbalance in <-, 135 then 136
stack imbalance in .Call, 159 then 160
stack imbalance in .Call, 160 then 161
stack imbalance in .Call, 161 then 162
stack imbalance in .Call, 162 then 163
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stack imbalance in .Call, 164 then 165
stack imbalance in .Call, 165 then 166
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stack imbalance in .Call, 583 then 584
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stack imbalance in .Call, 598 then 599
stack imbalance in .Call, 599 then 600
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stack imbalance in .Call, 636 then 637
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stack imbalance in .Call, 649 then 650
stack imbalance in .Call, 650 then 651
stack imbalance in .Call, 651 then 652
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stack imbalance in .Call, 654 then 655
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stack imbalance in .Call, 701 then 702
stack imbalance in .Call, 702 then 703
stack imbalance in .Call, 703 then 704
stack imbalance in .Call, 704 then 705
stack imbalance in .Call, 705 then 706
stack imbalance in .Call, 706 then 707
stack imbalance in .Call, 707 then 708
stack imbalance in .Call, 708 then 709
stack imbalance in .Call, 709 then 710
stack imbalance in .Call, 710 then 711
stack imbalance in .Call, 711 then 712
stack imbalance in .Call, 712 then 713
stack imbalance in .Call, 713 then 714
stack imbalance in .Call, 714 then 715
stack imbalance in .Call, 715 then 716
stack imbalance in .Call, 716 then 717
stack imbalance in .Call, 717 then 718
stack imbalance in .Call, 718 then 719
stack imbalance in .Call, 719 then 720
stack imbalance in .Call, 720 then 721
```

```
stack imbalance in .Call, 721 then 722
stack imbalance in .Call, 722 then 723
stack imbalance in .Call, 723 then 724
stack imbalance in .Call, 724 then 725
stack imbalance in .Call, 725 then 726
stack imbalance in .Call, 726 then 727
stack imbalance in .Call, 727 then 728
stack imbalance in .Call, 728 then 729
stack imbalance in .Call, 729 then 730
stack imbalance in .Call, 730 then 731
stack imbalance in .Call, 731 then 732
stack imbalance in .Call, 732 then 733
stack imbalance in .Call, 733 then 734
stack imbalance in .Call, 734 then 735
stack imbalance in .Call, 735 then 736
stack imbalance in .Call, 736 then 737
stack imbalance in .Call, 737 then 738
stack imbalance in .Call, 738 then 739
stack imbalance in .Call, 739 then 740
stack imbalance in .Call, 740 then 741
stack imbalance in .Call, 741 then 742
stack imbalance in .Call, 742 then 743
stack imbalance in .Call, 743 then 744
stack imbalance in .Call, 744 then 745
stack imbalance in .Call, 745 then 746
stack imbalance in .Call, 746 then 747
stack imbalance in .Call, 747 then 748
stack imbalance in .Call, 748 then 749
stack imbalance in .Call, 749 then 750
stack imbalance in .Call, 750 then 751
stack imbalance in .Call, 751 then 752
stack imbalance in .Call, 752 then 753
stack imbalance in .Call, 753 then 754
stack imbalance in .Call, 754 then 755
stack imbalance in .Call, 755 then 756
stack imbalance in .Call, 756 then 757
stack imbalance in .Call, 757 then 758
stack imbalance in .Call, 758 then 759
stack imbalance in .Call, 759 then 760
stack imbalance in .Call, 760 then 761
stack imbalance in .Call, 761 then 762
stack imbalance in .Call, 762 then 763
stack imbalance in .Call, 763 then 764
stack imbalance in .Call, 764 then 765
stack imbalance in .Call, 765 then 766
stack imbalance in .Call, 766 then 767
stack imbalance in .Call, 767 then 768
stack imbalance in .Call, 768 then 769
stack imbalance in .Call, 769 then 770
stack imbalance in .Call, 770 then 771
stack imbalance in .Call, 771 then 772
stack imbalance in .Call, 772 then 773
stack imbalance in .Call, 773 then 774
stack imbalance in .Call, 774 then 775
stack imbalance in .Call, 775 then 776
stack imbalance in .Call, 776 then 777
stack imbalance in .Call, 777 then 778
```

```
stack imbalance in .Call, 778 then 779
stack imbalance in .Call, 779 then 780
stack imbalance in .Call, 780 then 781
stack imbalance in .Call, 781 then 782
stack imbalance in .Call, 782 then 783
stack imbalance in .Call, 783 then 784
stack imbalance in .Call, 784 then 785
stack imbalance in .Call, 785 then 786
stack imbalance in .Call, 786 then 787
stack imbalance in .Call, 787 then 788
stack imbalance in .Call, 788 then 789
stack imbalance in .Call, 789 then 790
stack imbalance in .Call, 790 then 791
stack imbalance in .Call, 791 then 792
stack imbalance in .Call, 792 then 793
stack imbalance in .Call, 793 then 794
stack imbalance in .Call, 794 then 795
stack imbalance in .Call, 795 then 796
stack imbalance in .Call, 796 then 797
stack imbalance in .Call, 797 then 798
stack imbalance in .Call, 798 then 799
stack imbalance in .Call, 799 then 800
stack imbalance in .Call, 800 then 801
stack imbalance in .Call, 801 then 802
stack imbalance in .Call, 802 then 803
stack imbalance in .Call, 803 then 804
stack imbalance in .Call, 804 then 805
stack imbalance in .Call, 805 then 806
stack imbalance in .Call, 806 then 807
stack imbalance in .Call, 807 then 808
stack imbalance in .Call, 808 then 809
stack imbalance in .Call, 809 then 810
stack imbalance in .Call, 152 then 803
stack imbalance in {, 150 then 801
stack imbalance in <-, 144 then 795
stack imbalance in {, 142 then 793
stack imbalance in if, 140 then 791
stack imbalance in {, 138 then 789
stack imbalance in if, 136 then 787
stack imbalance in {, 133 then 785
stack imbalance in standardGeneric, 125 then 777
stack imbalance in <-, 119 then 771
stack imbalance in .Internal, 114 then 766
stack imbalance in {, 108 then 760
stack imbalance in .Internal, 99 then 751
stack imbalance in {, 96 then 748
stack imbalance in if, 94 then 746
stack imbalance in {, 92 then 744
stack imbalance in <-, 86 then 738
[1] 19.25  0.04 19.30  0.00  0.00
stack imbalance in {, 84 then 736
stack imbalance in if, 82 then 734
> summary(mB4)
Generalized linear mixed model fit using Laplace
Formula: use ~ urban + age + livch + (urban | district)
   Data: Contraception
 Family: binomial(logit link)
```

```
      AIC      BIC    logLik deviance
 2391.106 2368.837 -1199.553 2399.106
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 district (Intercept) 0.38774  0.62269
          urbanY      0.66745  0.81698  -0.793
# of obs: 1934, groups: district, 60

Estimated scale (compare to 1)  0.9759564

Fixed effects:
              Estimate Std. Error  z value  Pr(>|z|)
(Intercept) -1.6653580  0.1572532 -10.5903 < 2.2e-16
urbanY       0.7907018  0.1681257   4.7030 2.563e-06
age         -0.0258356  0.0079082  -3.2669  0.001087
livch1       1.0980461  0.1580051   6.9494 3.668e-12
livch2       1.3338218  0.1745854   7.6399 2.173e-14
livch3+      1.3214900  0.1795440   7.3603 1.836e-13

Correlation of Fixed Effects:
        (Intr) urbanY age    livch1 livch2
urbanY  -0.481
age      0.416 -0.036
livch1  -0.548  0.038 -0.211
livch2  -0.586  0.068 -0.378  0.487
livch3+ -0.695  0.062 -0.674  0.537  0.616
```

5 Growth curve model for repeated measures data

```
> str(Oxboys)
`data.frame':        234 obs. of  4 variables:
 $ Subject : Factor w/ 26 levels "1","10","11",..: 1 1 1 1 1 1 1 1 1 12 ...
 $ age     : num  -1.0000 -0.7479 -0.4630 -0.1643 -0.0027 ...
 $ height  : num  140 143 145 147 148 ...
 $ Occasion: Factor w/ 9 levels "1","2","3","4",..: 1 2 3 4 5 6 7 8 9 1 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 height ~ age | Subject
  .. .. ..- attr(*, ".Environment")=length 8 <environment>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  .. ..- attr(*, "source")= chr "function (x) max(x, na.rm = TRUE)"
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 2
  .. ..$ age   : chr "Centered age"
  .. ..$ height: chr "Height"
  ..$ units       :List of 1
  .. ..$ height: chr "(cm)"
> system.time(mX1 <- lmer(height ~ age + I(age^2) + I(age^3) +
+     I(age^4) + (age + I(age^2) | Subject), Oxboys), gc = TRUE)
[1] 0.19 0.00 0.19 0.00 0.00
> summary(mX1)
```

```
Linear mixed-effects model fit by REML
Formula: height ~ age + I(age^2) + I(age^3) + I(age^4) + (age + I(age^2) |      Subject)
   Data: Oxboys
      AIC     BIC    logLik MLdeviance REMLdeviance
 651.9081 693.372 -313.9541   625.3593     627.9081
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 64.03492 8.00218
          age          2.86417 1.69239  0.614
          I(age^2)     0.67429 0.82115  0.215 0.658
 Residual              0.21737 0.46623
# of obs: 234, groups: Subject, 26

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept) 149.01887    1.57037 229 94.8944 < 2.2e-16
age           6.17418    0.35650 229 17.3187 < 2.2e-16
I(age^2)      1.12823    0.35144 229  3.2103  0.001516
I(age^3)      0.45385    0.16246 229  2.7937  0.005653
I(age^4)     -0.37690    0.30018 229 -1.2556  0.210552

Correlation of Fixed Effects:
         (Intr) age    I(g^2) I(g^3)
age       0.572
I(age^2)  0.076  0.264
I(age^3) -0.001 -0.340  0.025
I(age^4)  0.021  0.016 -0.857 -0.021
> system.time(mX2 <- lmer(height ~ poly(age, 4) + (age + I(age^2) |
+     Subject), Oxboys), gc = TRUE)
[1] 0.2 0.0 0.2 0.0 0.0
> summary(mX2)
Linear mixed-effects model fit by REML
Formula: height ~ poly(age, 4) + (age + I(age^2) | Subject)
   Data: Oxboys
      AIC      BIC    logLik MLdeviance REMLdeviance
 640.8686 682.3324 -308.4343   625.3593     616.8686
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 64.03464 8.00217
          age          2.86418 1.69239  0.614
          I(age^2)     0.67429 0.82115  0.215 0.658
 Residual              0.21737 0.46623
# of obs: 234, groups: Subject, 26

Fixed effects:
              Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)   149.51976    1.59031 229 94.0194 < 2.2e-16
poly(age, 4)1  64.54095    3.32786 229 19.3941 < 2.2e-16
poly(age, 4)2   4.20322    1.02361 229  4.1063 5.597e-05
poly(age, 4)3   1.29077    0.46628 229  2.7682  0.006098
poly(age, 4)4  -0.58547    0.46630 229 -1.2556  0.210552

Correlation of Fixed Effects:
            (Intr) p(,4)1 p(,4)2 p(,4)3
poly(ag,4)1 0.631
poly(ag,4)2 0.230  0.583
poly(ag,4)3 0.000  0.000  0.000
poly(ag,4)4 0.000  0.000  0.000  0.000
```

6 Cross-classification model

```
> str(ScotsSec)

`data.frame':        3435 obs. of  6 variables:
 $ verbal : num  11 0 -14 -6 -30 -17 -17 -11 -9 -19 ...
 $ attain : num  10 3 2 3 2 2 4 6 4 2 ...
 $ primary: Factor w/ 148 levels "1","2","3","4",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ sex    : Factor w/ 2 levels "M","F": 1 2 1 1 2 2 2 1 1 1 ...
 $ social : num  0 0 0 20 0 0 0 0 0 0 ...
 $ second : Factor w/ 19 levels "1","2","3","4",..: 9 9 9 9 9 9 1 1 9 9 ...

> system.time(mS1 <- lmer(attain ~ sex + (1 | primary) + (1 |
+     second), ScotsSec), gc = TRUE)

[1] 0.28 0.01 0.29 0.00 0.00

> summary(mS1)

Linear mixed-effects model fit by REML
Formula: attain ~ sex + (1 | primary) + (1 | second)
   Data: ScotsSec
      AIC      BIC    logLik MLdeviance REMLdeviance
 17137.91 17168.62 -8563.956   17123.49     17127.91
Random effects:
 Groups   Name        Variance Std.Dev.
 primary  (Intercept) 1.10962  1.0534
 second   (Intercept) 0.36966  0.6080
 Residual             8.05511  2.8382
# of obs: 3435, groups: primary, 148; second, 19

Fixed effects:
             Estimate Std. Error   DF t value  Pr(>|t|)
(Intercept) 5.2552e+00 1.8432e-01 3433 28.5108 < 2.2e-16
sexF        4.9851e-01 9.8255e-02 3433  5.0737 4.109e-07

Correlation of Fixed Effects:
     (Intr)
sexF -0.264
```