tency), *mono* means that the node has exactly one model up to isomorphism (i.e. it is monomorphic), and *def* means that the node has exactly one model (the latter will occur only rarely).

4.3 Translating Development Graphs along Institution Comorphisms

Given a model-isomorphic simple theoroidal institution comorphism $R = (\Phi, \alpha, \beta)\colon I \to J$, we can extend this comorphism to a translation of development graphs over I into development graphs over J in the following way:

Given a development graph $\mathcal{DG}$ over I, let $R(\mathcal{DG})$ have the same nodes and links as $\mathcal{DG}$ (for clarity, given a node $N \in \mathcal{DG}$, we call the corresponding node $R(N) \in R(\mathcal{DG})$, and similarly for definition links). The associated signatures, local axioms and signature morphisms differ, of course:

- if $N \in \mathcal{DG}$, then $\Sigma^{R(N)} = Sig(\Phi(\Sigma^N))$, and
$$\Psi^{R(N)} = \alpha_{\Sigma^N}(\Psi^N) \cup Ax(\Phi(\Sigma^N))$$
- the signature morphisms decorating a link L are translated along Φ, and intermediate signatures Σ are replaced with $Sig(\Phi(\Sigma))$, yielding a link $R(L)$.

Theorem 4.14. *Given a model-isomorphic simple theoroidal institution comorphism $R = (\Phi, \alpha, \beta)\colon I \to J$ and a development graph $\mathcal{DG}$ over I, for each $N \in \mathcal{DG}$, the isomorphism*

$$\beta_{\Sigma^N} : \mathbf{Mod}(\Sigma^N) \to \mathbf{Mod}(\Phi(\Sigma^N))$$

restricts to the isomorphism

$$\beta_{\Sigma^N} : \mathbf{Mod}(N) \to \mathbf{Mod}(R(N))$$

Proof. First, note that indeed $\mathbf{Mod}(R(N)) \subseteq \mathbf{Mod}(\Phi(\Sigma^N))$, because $\Psi^{R(N)}$ includes $Ax(\Phi(\Sigma^N))$. We now proceed by induction over $\mathcal{DG}$. Hence, it suffices to show for each $M \in \mathbf{Mod}(\Phi(\Sigma))$:

[6] We here assume that the empty signature is initial.
[7] Here we tacitly assume that there is some special node having the initial signature and the empty set of axioms.

1. $\beta_{\Sigma^N}(M) \models \Psi^N$ iff $M \models \Psi^{R(N)}$,
2. for any ingoing definition link L into N, $\beta_{\Sigma^N}(M)$ satisfies L iff M satisfies $R(L)$.

Both can be shown in a straightforward way, using the satisfaction condition of the comorphism, naturality and isomorphism property of β and the fact that for any I-signature morphism σ, $\Phi(\sigma)$ is a theory morphism. □

Theorem 4.15. *Given a model-isomorphic simple theoroidal institution comorphism $R = (\Phi, \alpha, \beta)\colon I \to J$ and a development graph $\mathcal{DG}$ over I, let L be a theorem link over $\mathcal{DG}$. Then*

$$\mathcal{DG} \models L \text{ iff } R(\mathcal{DG}) \models R(L)$$

Proof. By Theorem 4.14 and Remark 4.13. □

Note that with this translation of development graphs along comorphisms, new local axioms coming from $Ax(\Phi(\Sigma^N))$ are often partly repeated. One can optimize this by adding at each node only those axioms from $Ax(\Phi(\Sigma^N))$ that are not already present via links from other nodes.

4.4 Proof Rules for Development Graphs

In this section, we introduce logic-independent proof rules for development graphs. These rely on a logic-specific entailment relation for basic specifica-
tions as introduced in Chap. 1, as well as on logic-specific proof rules for